DAR model notes

**The DAR(1) model**

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Normal form,

Expanded form,

**Forecasting**

Since the DAR model contains an ARCH process, an analytical solution of the -step forecast is only obtainable for (also ???). This is done using forward recursion. Longer forecast horizons than cannot be derived analytically since XXX. But can be obtained using Monte Carlo (bootstrap) methods. We will only consider -step forecast.

Let denote the information set at time . Assuming that the random innovation follows a standard normal Gaussian (maybe student-t???), then and , as is not contained in . The optimal 1-step forecast is given by the conditional mean,

Since is contained in the information set, . Also, is independent of , so the expectation over the product is equal to the product of the two expectations,

Since is contained in the information set, ,

Theoretically, in expectation. However, in practice can be modelled auxiliareliry as a Moving Average (MA) process based on the lagged realised innovations. Let be a MA() process,

Then the -step forecast of the MA() process can be derived as,

Since is not contained in the information set, then . The past innovations are observed and can be derived by filtering the estimated DAR() residuals,, with the fitted ARCH() process,, such that . The auxiliary forecast of the innovation process becomes,

By assumption . However, the MA() structure might be able to absorb unwanted autocorrelation in the innovation process. Therefore, the -step forecast of the DAR() model, , is given by,

**DAR-MA(1)**:

Together,

Assuming MA(1),

Correspondonly for ,

One 1-step forecast,

Given is not contained in the information set , our best guess of the expected value is . Thus,

Since ,

Since , then by definition,

Inserting yields an expression of the 1-step forecast solely as a funciton of and ,

**New framework**

**The DAR-MA(1)**:

The general form,

Assuming DAR-MA(1):

Using that and therefore ,

Redefining to get,

**Conditional mean**

Therefore,

**Conditional variance**

The conditional variance can be decomposed as,

We start with ,

Using that and we can reduce the expression to,

Therefore,

Now for ,

Now ,

This reduces to,

**The new estimation error**

We have that,

Defining the new error,

The new error can thus be defined as,

With,

Thus,

**Distribution**

We have derived that is conditionally distributed as,

Assuming that , the conditional distribution of is then,

As depends on both and , the model violates the Markov-chain assumption assumptin I.3.1 (i) in ARCH Part 1. However, we can reformulate the model on companion form as,

Using this formulation, conditional on the past values , depends only on as shown,

However, violates the assumptin I.3.1 (ii) in ARCH Part 1, as it is singular. This can be shown by using the formula for conditional densities , yielding,

As shown before, the first term is is continous Gaussian density. However, , since is fixed (already in the information set). Therefore, is no longer a random variable, and the conditional density function becomes a Dirac delta function at , which is not continues, violating the assumption I.3.1 (ii).

The problem of singularity can be solved by instead conditioning on ,

Since is a continious Gaussian density, it follows that is also. It holds that the product of two continous Gaussian densities is also a continous Gaussian density, therefore satisfies assumption I.3.1 (ii).

**Log-likelihood function**

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Ignoring all constants,

**Staionarity**

ARCH(2)

We have that,

Now define with structure similar a VAR(1) model,

Then,

It can be shown that is stationary (fractional moments) provided that,

Note, for any VAR(1) process,

The process is stationary if with denoting the absolute value of the largest eigenvalue of matrix . This holds because, as .

DAR(1)-MA(1):

We have that,

**Drift criterion**

We consider the drift function

Let,

We consider the drift function

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**New new idea…**

Let,

For ,

Alternative derivation,

Inserting ,

We have the following relationship between and ,

Insering ,

Inserting

Noticing a reccurring structure,

We can now rewrite the model as,

What a

With forecast,

With given as,